## Chapter 10

**Statistical Inference About Means and Proportions with Two Populations**

**Learning Objectives**

1. Be able to develop interval estimates and conduct hypothesis tests about the difference between two population means whenandare known.

2. Know the properties of the sampling distribution of .

3. Be able to use the *t* distribution to conduct statistical inferences about the difference between two population means whenandare unknown.

4. Learn how to analyze the difference between two population means when the samples are independent and when the samples are matched.

5. Be able to develop interval estimates and conduct hypothesis tests about the difference between two population proportions.

6. Know the properties of the sampling distribution of .

**Solutions:**

1. a.  = 13.6 - 11.6 = 2

b. 1.645





2  .98 (1.02 to 2.98)

c. 1.96



2  1.17 (.83 to 3.17)

2. a. 

b. *p*-value = 1.0000 - .9788 = .0212

c. *p*-value  .05, reject *H*0.

3. a. 

b. *p*-value = 2(.0630) = .1260

c. *p*-value > .05, do not reject *H*0.

4. a. = population mean for smaller cruise ships

= population mean for larger cruise ships

= 85.36 – 81.40 = 3.96

b. 



c. 3.96 ± 1.88 (2.08 to 5.84)

5. a. = 135.67 – 68.64 = 67.03

b. 

c. 67.03  17.08 (49.95 to 84.11) We estimate that men spend $67.03 more

than women on Valentine’s Day with a margin of error of $17.08.

6. = mean hotel price in Atlanta

= mean hotel price in Houston

*H*0: 

*H*a: 



*p*-value = .0351

*p*-value  .05; reject *H*0. The mean price of a hotel room in Atlanta is lower than the mean price of a hotel room in Houston.

7. a. = population mean satisfaction score for Target customers

= population mean satisfaction score for Walmart customers

*H*0: 

*H*a: 

b. = 79 – 71 = 8



For this two-tailed test, *p*-value is two times the upper-tail area at *z* = 2.46.

*p*-value = 2(1.0000 – .9931) = .0138

*p*-value .05; reject *H*0. The population mean satisfaction scores differ for the two retailers.

c. 



8  6.37 (1.63 to 14.37)

Target shows a higher population mean customer satisfaction score than Walmart with the 95% confidence interval indicating that Target has a population mean customer satisfaction score that is 1.63 to 14.37 higher than Walmart.

8. a. This is an upper tail hypothesis test.

*H*0:

*H*a:



*p*-value = area in upper tail at *z* = 2.74

*p*-value = 1.0000 - .9969 = .0031

Since .0031  α = .05, we reject the null hypothesis. The difference is significant. We can conclude that customer service has improved for Rite Aid.

b. This is another upper tail test but it only involves one population.

*H*0:

*H*a:



*p*-value = area in upper tail at *z* = .39

*p*-value = 1.0000 - .6517 = .3483

Since .3483 > α = .05, we cannot reject the null hypothesis. The difference is not statistically significant.

c. This is an upper tail test similar to the one in part (a).

*H*0:

*H*a:



*p*-value = area in upper tail at *z* = 1.83

*p*-value = 1.0000 - .9664 = .0336

Since .0336  α = .05, we reject the null hypothesis. The difference is significant. We can conclude that customer service has improved for Expedia.

d. We will reject the null hypothesis of “no increase” if the *p*-value ≤ .05. For an upper tail hypothesis test, the *p*-value is the area in the upper tail at the value of the test statistic. A value of *z* = 1.645 provides an upper tail area of .05. So, we must solve the following equation for .





This tells us that as long as the 2008 score for a company exceeds the 2007 score by 1.80 or more the difference will be statistically significant.

e. The increase from 2007 to 2008 for J.C. Penney is not statistically significant because it is less than 1.80. We cannot conclude that customer service has improved for J.C. Penney.

9. a. = 22.5 - 20.1 = 2.4

b. 

Use *df* = 45.

c. *t*.025 = 2.014



d. 2.4 2.1 (.3 to 4.5)

10. a. 

b. 

Use *df* = 65

c. Using *t* table, area in tail is between .01 and .025

two-tail *p*-value is between .02 and .05.

Exact *p*-value corresponding to *t* = 2.18 is .0329

d. *p*-value .05, reject *H*0.

11. a.  

b. 



c. = 9 - 7 = 2

d. 

Use *df* = 9, *t*.05 = 1.833



2 2.17 (-.17 to 4.17)

12. a. = 22.5 - 18.6 = 3.9

b. 

Use *df* = 87, *t*.025 = 1.988



3.9  (.6 to 7.2)

13. a. 







b. = 42.5 – 22.3 = 20.2 or $20,200

The mean annual cost to attend private colleges is $20,200 more than the mean annual cost to attend public colleges.

c. 

Use *df* = 14, *t*.025 = 2.145





20.3  5.5 (14.8 to 25.8)

95% confidence interval, private colleges have a population mean annual cost $14,800 to $25,800 more expensive than public colleges.

14. a. *H*0: 

*H*a: 

b. 

c. = 87.55

Rounding down, we will use a *t* distribution with 87 degrees of freedom. From the *t* table we see that *t* = -2.41 corresponds to a *p*-value between .005 and .01.

Exact *p*-value corresponding to *t* = -2.41 is .009.

d. *p*-value .05, reject *H*0. We conclude that the salaries of staff nurses are lower in Tampa than in Dallas.

15. a. **1 = population mean annual lease rate per square meter in Hong Kong

**2 = population mean annual lease rate per square meter in Paris

*H*0: 

*H*a: 

b. = $1114 - $989 = $125 per square meter





Use *df* = 56

Using *t* table, *p*-value is between .005 and .01.

Exact *p*-value corresponding to *t* = 2.40 is .0099

*p*-value .01, reject *H*0. Conclusion: The annual lease rate in Hong Kong is significantly higher than in Paris.

16. a.  = population mean verbal score parents college grads

= population mean verbal score parents high school grads

*H*0: 

*H*a: 

b. 



= 525 - 487 = 38 points higher if parents are college grads

c. 







Use *df* = 25

Using *t* table, *p*-value is between .025 and .05

Exact *p*-value corresponding to *t* = 1.80 is .0420

d. *p*-value .05, reject *H*0. Conclude higher population mean verbal scores for students whose parents are college grads.

17. a. *H*0: 

*H*a: 

b. 

c. 

Use *df* = 16

Using *t* table, *p*-value is between .025 and .05

Exact *p*-value corresponding to *t* = 1.99 is .0320

d. *p*-value .05, reject *H*0. The consultant with more experience has a higher population mean rating.

18. a. Let = population mean minutes late for delayed AirTran flights

= population mean minutes late for delayed Southwest flights

*H*0: 

*H*a: 

b. minutes

minutes

The difference between sample mean delay times is 50.6 – 52.8 = -2.2 minutes, which indicates the sample mean delay time is 2.2 minutes less for AirTran Airways.

c. Sample standard deviations: *s*1 = 26.57 and *s*2 = 20.11





Use *df* = 42

*p*-value for this two-tailed test is two times the lower-tail area for *t* = -.32.

Using *t* table, *p*-value is greater than 2(.20) = .40

Exact *p*-value corresponding to *t* = -.32 with 42 *df* is .7506

*p*-value > .05, do not reject *H*0. We cannot reject the assumption that the population mean delay times are the same at AirTran Airways and Southwest Airlines. There is no statistical evidence that one airline does better than the other in terms of their population mean delay time.

19. a. 1, 2, 0, 0, 2

b. 

c. 

d. 

*df* = *n* - 1 = 4

Using *t* table, *p*-value is between .025 and .05

Exact *p*-value corresponding to *t* = 2.24 is .0443

Reject *H*0; conclude **d > 0.

20. a. 3, -1, 3, 5, 3, 0, 1

b. 

c. 

d. = 2

e. With 6 degrees of freedom *t*.025 = 2.447



2  1.93 (.07 to 3.93)

21. Difference = rating after - rating before

*H*0: **d  0

*H*a: **d > 0

= .625 and = 1.30



*df* = *n* - 1 = 7

Using *t* table, *p*-value is between .10 and .20

Exact *p*-value corresponding to *t* = 1.36 is .1080

Do not reject *H*0; we cannot conclude that seeing the commercial improves the mean potential to purchase.

22. a. Let *di* = 1st quarter price per share – beginning of year price per share



b. 

With *df* = 24, *t*.025 = 2.064



3.41 ± 2.064 

Confidence interval: $3.41 ± $1.74 ($1.67 to $5.15)

The 95% confidence interval shows that the population mean price per share of stock has increased between $1.67 and $5.15 over the three-month period.

Note that at the beginning of year



With this as the sample mean price per share of stock at the beginning of 2012, the confidence interval ($1.67 to $5.15) indicates the percentage change in the population mean price per share of stock would have increased from

1.67/46.13 = .036, or 3.6%

to 5.15/46.13 = .112, or 11.2%

Thus, for the population of stocks, the mean price per share has increase between 3.6% and 11.2% over the three-month period. This was excellent news for the 1st quarter of 2012. Stock prices were having one of the largest quarterly increases in years. The outlook for a recovering economy was very good at the end of the 1st quarter of 2012.

23. a. **1 = population mean grocery expenditures

**2 = population mean dining-out expenditures

*H*0: 

*H*a: 

b. 

*df* = *n* - 1 = 41

*p*-value0

Conclude that there is a difference between the annual population mean expenditures for groceries and for dining-out.

c. Groceries has the higher mean annual expenditure by an estimated $850.





850 ± 350 (500 to 1200)

24. a. Difference = Current Year Airfare – Previous Year Airfare

*H*0: **≤ 0

*H*a: ** > 0

Differences 30, 63, -42, 10, 10, -27, 50, 60, 60, -30, 62, 30







*df* = *n* - 1 = 11

Using *t* table, *p*-value is between .05 and .025

Exact *p*-value corresponding to *t* = 2.05 is .0325

Since *p*-value < .05, reject *H*0. We can conclude that there has been a significance increase in business travel airfares over the one-year period.

b. Current year: 

Previous year: 

c. One-year increase = $487 - $464 = $23

$23/$464 = .05, or a 5% increase in business travel airfares for the one-year period.

25. a. Difference = math score – writing score

*H*0: **= 0

*H*a: ** ≠ 0

Use difference data: 66, 52, 65, -38, 28, -24, 50, 40, -5, 31, 55, -20







*df* = *n* - 1 = 11

Using *t* table, lower-tail area is between .025 and .01.

Thus, the two-tailed test *p*-value is between .05 and .02.

Exact *p*-value corresponding to *t* = -2.34 is .0392

*p*-value .05, reject *H*0. Conclude that there is a significant difference between the population mean scores for the SAT math test and the SAT writing test.

b. 

for the math test

for the writing test

The SAT math test has a higher mean score than the SAT writing test.

26. a. *H*0: **d= 0

*H*a: **d≠ 0

Differences: -2, -1, -5, 1, 1, 0, 4, -7, -6, 1, 0, 2, -3, -7, -2, 3, 1, 2, 1, -4







*df* = *n* – 1 = 19

Using *t* table, area in tail is between .05 and .10

Two-tail *p*-value must be between .10 and .20

Exact *p*-value corresponding to *t* = -1.42 is .1718

Cannot reject *H*0. There is no significant difference between the mean scores for the first and fourth rounds.

b. = -1.05; First round scores were lower than fourth round scores.

c. *α* = .05 *df* = 19 *t* = 1.729

Margin of error = = 

Yes, just check to see if the 90% confidence interval includes a difference of zero. If it does, the difference is not statistically significant.

90% Confidence interval: -1.05 ± 1.28 (-2.33, .23)

The interval does include 0, so the difference is not statistically significant.

27. a. Difference = Price deluxe - Price Standard

*H*0: **d = 10

*H*a: **d  10

= 8.86 and = 2.61



*df* = *n* - 1 = 6

Using *t* table, area is between .10 and .20

Two-tail *p*-value is between .20 and .40

Exact *p*-value corresponding to *t* = -1.16 is .2901

Do not reject *H*0; we cannot reject the hypothesis that a $10 price differential exists.

b. 95% Confidence interval





or (6.45 to 11.27)

28. a. = .48 - .36 = .12

b. 



.12  .0614 (.0586 to .1814)

c. 

.12  .0731 (.0469 to .1931)

29. a. 



*p* - value = 1.0000 - .9554 = .0446

b. *p*-value .05; reject *H*0.

30. = 220/400 = .55 = 192/400 = .48





.07  .0691 (.0009 to .1391)

7% more executives are predicting an increase in full-time jobs. The confidence interval shows the difference may be from 0% to 14%.

31. a. Professional Golfers: = 688/1075 = .64

Amateur Golfers: = 696/1200 = .58

Professional golfers have the better putting accuracy.

b. 

Professional golfers make 6% more 6-foot putts than the very best amateur golfers.

c. 



.06  .04 (.02 to .10)

The confidence interval shows that professional golfers make from 2% to 10% more 6-foot putts than the best amateur golfers.

32. a. 



b.  = 300/811 = .3699 37% of women would ask directions

c.  = 255/750 = .3400 34% of men would ask directions

d. 



Upper tail *p*-value is the area to the right of the test statistic

Using normal table with *z* = 1.23: *p*-value = 1 - .8907 = .1093

*p*-value > *α* ; do not reject 

We cannot conclude that women are more likely to ask directions.

33. Let *p*1 = the population proportion of delayed departures at Chicago O’Hare

*p*2 = the population proportion of delayed departures at Atlanta Hartsfield-Jackson

a. *H*0: *p*1 - *p*2 = 0

*H*a: *p*1 - *p*2 ≠ 0

b.  = 252/900 = .28

c.  = 312/1200 = .26

d. 



*p*-value = 2(1 - .8461) = .3078

Do Not Reject *H*0. We cannot conclude that there is a difference between the proportion of delayed departures at the two airports.

34. a.  = 192/300 = .64

b. = 117/260 = .45

c. = .64 - .45 = .19





.19  .0813 (.1087 to .2713)

35. a. *H*0: *p*1 - *p*2 = 0

*H*a: *p*1 - *p*2  0

 = 63/150 = .42

 = 60/200 = .30





*p*-value = 2(1.0000 - .9901) = .0198

*p*-value.05, reject *H*0. There is a difference between the recall rates for the two commercials.

b. 



.12  .1014 (.0186 to .2214)

Commercial A has the better recall rate.

36. a. Let = population proportion of rooms occupied for current year

= population proportion of rooms occupied for previous year

*H*0: **

*H*a: **

b. = 1470/1750 = .84 (current year)

= 1458/1800 = .81 (previous year)

c. 



*p*-value is are in the upper tail at *z* = 2.35

*p*-value = 1.0000 - .9906 = .0094

*p*-value.05, reject *H*0. There has been an increase in the hotel occupancy rate.

d. 



.03  .025 (.005 to .055)

Officials would likely be pleased with the occupancy statistics. The trend for the current year is an increase in hotel occupancy rates compared to last year. The point estimate is a 3% increase with a 95% confidence interval from .5% to 5.5%.

37. a. Let = population proportion of men expecting to get a raise or promotion this year

= population proportion of women expecting to get a raise or promotion this year

*H*0: *p*1 - *p*2 < 0

*H*a: *p*1 - *p*2 > 0

b.  = 104/200 = .52 (52%)

 = 74/200 = .37 (37%)

c. 



*p*-value = 1.0000 - .9987 = .0013

Reject *H*0. There is a significant difference between the population proportions with a great proportion of men expecting to get a raise or a promotion this year.

38. *H*0: **1 - **2 = 0

*H*a: **1 - **2  0



*p*-value = 2(1.0000 - .9974) = .0052

*p*-value .05, reject *H*0. A difference exists with system B having the lower mean checkout time.

39. a.  Mean resale price in 2006

 Mean resale price in 2009

Difference = 225,897 – 170,993 = 54,904

Using sample mean prices, the 2009 resale prices are $54,904 less than in 2006.

b. 





Use *df* = 54, *t*.005 = 2.670





54904  32931 (21,973 to 87,835)

We are 99% confident that home prices have declined by between $21,973 and $87,835.

c. To answer this question we need to conduct a one-tailed hypothesis test. No value for the level of significance (α) has been given. But, most people would agree that a *p*-value  .01 would justify concluding that prices have declined from 2006 to 2009.



**



For *t* = 4.45 and *df* =54, we find *p*-value 0.00. Thus, we are justified in concluding that existing home prices have declined between 2006 and 2009.

40. a. 

**

b. *n*1 = 30 *n*2 = 30

= 16.23 = 15.70

*s*1 = 3.52 *s*2 = 3.31





Use *df* = 57

Using *t* table, *p*-value is greater than .20

Exact *p*-value corresponding to *t* = .60 is .2754

*p*-value > .05, do not reject *H*0. Cannot conclude that the mutual funds with a load have a greater mean rate of return.

41. a. *n*1 = 10 *n*2 = 8

= 21.2 = 22.8

*s*1 = 2.70 *s*2 = 3.55

= 21.2 - 22.8 = -1.6

Kitchens are less expensive by $1600.

b. 

Use *df* = 12, *t*.05 = 1.782



-1.6  2.7 (-4.3 to 1.1)

42. a.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| January 1 | April 30 |  |  |  |
| 10.13 | 12.21 | -2.08 | -4.53 | 20.5209 |
| 28.33 | 25.48 | 2.85 | 0.40 | 0.1600 |
| 73.97 | 66.10 | 7.87 | 5.42 | 29.3764 |
| 16.30 | 19.32 | -3.02 | -5.47 | 29.9209 |
| 45.27 | 43.05 | 2.22 | -0.23 | 0.0529 |
| 16.88 | 15.46 | 1.42 | -1.03 | 1.0609 |
| 2.29 | 5.98 | -3.69 | -6.14 | 37.6996 |
| 16.20 | 12.65 | 3.55 | 1.10 | 1.2100 |
| 59.83 | 52.36 | 7.47 | 5.02 | 25.2004 |
| 31.53 | 33.00 | -1.47 | -3.92 | 15.3664 |
| 19.44 | 20.26 | -0.82 | -3.27 | 10.6929 |
| 17.73 | 19.34 | -1.61 | -4.06 | 16.4836 |
| 17.71 | 13.36 | 4.35 | 1.90 | 3.6100 |
| 43.51 | 36.18 | 7.33 | 4.88 | 23.8144 |
| 61.82 | 49.44 | 12.38 | 9.93 | 98.6049 |
|  | Sum | 36.75 |  | 313.7742 |
|  |  |  |  |  |

 The mean price per share declined $2.45 over the four months.

b. 

*df* = *n* - 1 = 14, *t*.05 = 1.761

 = 

2.45 ± 2.15 ($.30 to $4.60)

We are 90% confident that the population mean price per share has decreased between $.30 and $4.60 over the four month period.

c. Sample mean price per share January 1: 

Percentage decrease over the 4 months: 

d. Mean price per share December 31, 2009 = $30.73(.92)(.92)(.92) = $23.93. This is a decline of

$30.73 – 23.93 = $6.80 per share for the year.

43. a. Let *p*1 = population proportion saying financial security more than fair in 2012

*p*2 = population proportion saying financial security more than fair in 2010

**

b.  = 410/1000 = .41 (41%)

 = 315/900 = .35 (35%)

c. 



*p*-value for this two-tailed test is two times the area in the upper tail at *z* = 2.69

*p*-value = 2(1.0000 - .9964) = .0072

*p*-value  .05, reject *H*0. Conclude the population proportions are not equal. There has been a change in the population proportion saying that their financial security is more than fair.

d. 



.06  .0436

95% Confidence Interval (.0164 to .1036)

e. Yes. Based on the results, the population proportion of adults saying that their financial security is more than fair has increased between 1.64% and 10.36% over the two-years.

44. a. = 76/400 = .19

= 90/900 = .10





*p*-value ≈ 0

Reject *H*0; there is a difference between claim rates.

b. 



.09  .0432 (.0468 to .1332)

Claim rates are higher for single males.

45. = 9/142 = .0634

= 5/268 = .0187





*p*-value = 2(1.0000 - .9911) = .0178

*p*-value .02, reject *H*0. There is a significant difference in drug resistance between the two states. New Jersey has the higher drug resistance rate.

46. a. March 2007: = 70/200 = .35

March 2008: = 70/150 = .47

b. 



Confidence interval: .12 ± 1.96(.0529) or .12 ± .1037 (.0163 to .2237)

c. Since the confidence interval in part (b) does not include 0, conclude that occupancy rates are higher in the first week of March 2008 than in the first week of March 2007. On the basis of this, expect occupancy rates to be higher for March 2008 than for March 2007.

47.  Most recent week

 One Week Ago

 One Month Ago

a. Point estimate =

Margin of error: 

95% confidence interval: -.211 ± .085 (-.296, -.126)

b. *H*0: *p*1 – *p*3 ≥ 0

*H*a: *p*1 – *p*3 < 0

c. 





*p*-value = .0025

With *p*-value ≤ .01, we reject *H*0 and conclude that bullish sentiment has declined over the past month.